S282 Astronomy

Redshifts, distances and lookback times

Study time: 45 minutes

Summary

In this activity you will use a piece of software called the *Cosmological modeller* in order to investigate the relationships between distance and redshift, and between lookback time and redshift, in a variety of cosmological models.

You should have read to the end of Chapter 5 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- Understand the Hubble expansion of the Universe.
- Understand the concept of mathematical modelling.
- Understand the process of testing a model by comparing predictions of the model with observed results.
- Understand the influence of certain cosmological parameters on the evolution and development of the Universe.

Background to the activity

In Chapter 5 of *An Introduction to Galaxies and Cosmology* you learned about the concept of modelling the Universe, using mathematical models based on Einstein's general theory of relativity and on principles derived from observation, such as the homogeneity and isotropy of the Universe and the Hubble expansion. Typically, such a model will involve a number of cosmological parameters such as the Hubble constant H_0 and the densities of matter and of dark energy.

You were introduced to one such class of models known as the Friedmann–Robertson–Walker (FRW) models. These are based on the Friedmann equation which determines how the scale factor of a model universe evolves over time, and describe a universe that is both homogeneous and isotropic. Although the basic equations of the FRW model do not change, the input parameters can be varied, leading to a class of closely related models. The variations between these models lead to different predictions which can then be compared to observation.

This process of modelling and comparison with observations allows us to determine the values of the input parameters that correspond most closely to those in the actual Universe. As you will learn in Chapter 7 of *An Introduction to Galaxies and Cosmology*, our present best estimate of the values of these parameters comes from the measurements made by the Wilkinson Microwave Anisotropy Probe (WMAP). The FRW model which adopts these parameter values is referred to in this activity as the 'WMAP model'.

WEB 00632 9

The *Cosmological modeller* program used in this activity consists of two sections: the first uses the FRW class of models to calculate the relationship between the redshift of a source and a number of important properties such as lookback time (defined below), recession speed and distance. It is this part of the program that we shall use in this activity. (The second section, which calculates power spectra of the CMB, is used in a later activity.)

Lookback time is defined as follows: suppose that the light from an object (with redshift z) is observed today, i.e. at time t_{obs}. The light from this object has been travelling through the expanding Universe since the time that it was emitted, t_{em}. The lookback time is defined as the difference between the times of emission and observation ($t_{\mathrm{obs}} - t_{\mathrm{em}}$).

The activity

1 Introduction

This activity concentrates on the relationships between redshift and lookback times, recession speeds and distances. In particular, we shall try to understand some of these relationships as they apply to the WMAP model of the Universe and to the 'critical model' that you met in Chapter 5.

■ What are the values of the curvature parameter k and the cosmological constant Λ in the critical model?

❑ In the critical model, $k = 0$ (i.e. the model is spatially flat) and the cosmological constant is zero ($\Lambda = 0$).

Let's think about observing a distant galaxy. It seems natural to ask how far away it is, or how long the light from it has been travelling, or even how fast it is receding from us. However, none of these quantities can be measured directly. The one quantity that can actually be measured (from the object's spectrum) is the redshift z. In this activity, then, all of these other quantities are calculated as a function of the redshift. The exact form of each relationship will depend on the parameters used to specify the model.

■ When describing distant objects, why is it preferable to refer to their redshifts as opposed to their distances?

❑ Since the redshift of an object is a *measured* quantity, it is independent of any mathematical model. The distance is an *implied* quantity whose value will depend on the model used. Any mention of distance should really be qualified by specifying the model. It is safer and less ambiguous to refer to the redshift, since it is a quantity that is independent of the assumed cosmological model.

2 Using the Cosmological modeller program

To start the program:

- Start the S282 Multimedia guide, and open the folder called 'Cosmology', then click on the icon for Redshifts, distances and lookback times
- Click Start to open the *Cosmological modeller* program.
- Click on the Redshifts, distances and lookback times button to open the lookback calculator section of the program.

Before starting to use the program, note that the density parameters that the program works with are all the present-day values of these parameters. So strictly speaking, these parameters should all be written in the form that explicitly shows that they relate to the present time, such as $\Omega_m(t_0)$ or $\Omega_{m,0}$. For brevity the program (and these notes) displays the density parameters without the time t_0 (or the subscript 0). However, as you work thought this activity it should be borne in mind that it is the present-day values of these parameters that are referred to.

The lookback calculation screen is divided into a number of sections.

In the lower left-hand corner is a box where you can enter values for the three parameters Ω_m, Ω_Λ and H_0 (see Figure 1).

When you first start the program, the values are set as follows:

$\Omega_m = 0.27$

$\Omega_\Lambda = 0.73$

$H_0 = 72$ km s^{-1} Mpc^{-1}

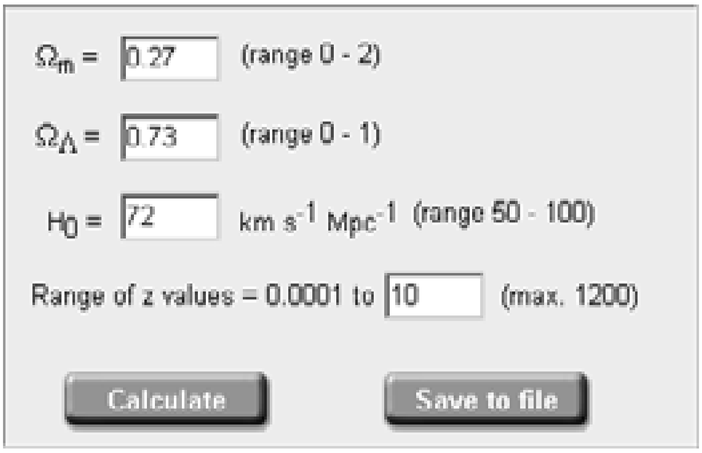

Figure 1 The lookback calculator.

As will be explained in Section 7.5.4 of *An Introduction to Galaxies and Cosmology*, these values represent the best fit with both the WMAP observational data, and with the measurements of very distant Type Ia supernovae. Notice that, in this case, $\Omega_m + \Omega_\Lambda = 1.00$.

■ What can be inferred about the curvature of an FRW model in which $\Omega_m + \Omega_\Lambda = 1$?

❑ If $\Omega_m + \Omega_\Lambda = 1$, then the curvature parameter $k = 0$, i.e. the model has zero curvature. (See *An Introduction to Galaxies and Cosmology*, Question 5.9. Note also that it has been assumed that any contribution to the total density from radiation is negligible.)

We will start by investigating the properties of this WMAP model, and you should leave the parameters set to these default values for the exercises described in Sections 3 to 7 below.

3 Age of the Universe

Perhaps the most fundamental prediction that we require of any model is an estimate of the overall age of the Universe. Before making calculations with the WMAP model, let's get a sense of scale by considering for a moment the prediction of the simpler 'critical model'. You may recall that there is a simple relationship between the Hubble constant and the age of a critical model universe. Consequently, as the following question illustrates, we don't need to use the *Cosmological modeller* to carry out this calculation.

Question 1

According to the critical model of the Universe described in Section 5.4.3, the age of the Universe is two-thirds of the Hubble time. Given a value of the Hubble constant of 72 km s^{-1} Mpc^{-1}, calculate the Hubble time ($1/H_0$), and hence the age of a critical model Universe with this value of H_0. Is this age consistent with observations of the ages of stars in globular clusters?

(Note that 1 year = 3.16×10^7 s, and 1 Mpc = 3.09×10^{22} m.)

You should now compare this result with the prediction of the WMAP model. To do this, you will need to use the *Cosmological modeller*.

- Leaving the model parameters set to their default values as shown above, press the Calculate button.

A graph will appear on the right-hand side of the screen. However, for the moment we are interested in the calculated value for the age of the Universe. This is displayed in the area to the right of the data entry box, below the graph.

- ■ How does the age of the Universe calculated by the program compare with the Hubble time and with the age of the critical model universe calculated in Question 1?
- ❑ The value calculated *for this model* by the *Cosmological modeller* (i.e. as specified by the combination of input parameters) is very close to the Hubble time – about 13.5 billion years. This is substantially greater than the age of the critical model (which is 2/3 of the Hubble time).

So, the age of the WMAP model is 50% greater than the age of the critical model (assuming a value of H_0 = 72 km s^{-1} Mpc^{-1}). Up until the early 1990s, the critical model was the model of choice for most astronomers and cosmologists. However, the relatively short age of the critical model universe was found to be inconsistent with the ages of the oldest stars in the Universe. The greater age (for a given value of H_0) of an accelerating model such as the WMAP model, removes this inconsistency.

4 Lookback time as function of redshift

Now take a look at the graph displayed at the upper right portion of the screen. This should be a plot of lookback time against redshift. If not, click on the arrow in the Choose graph box at the top left and select the first option: Lookback time v z (see Figure 2).

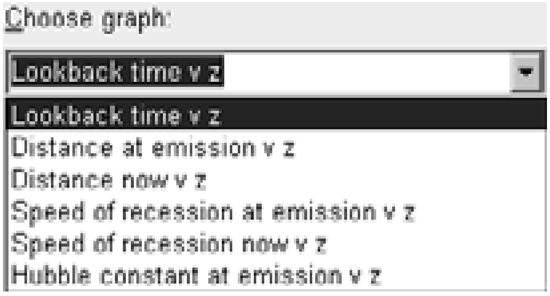

Figure 2 Selecting an option in the Choose graph box.

This graph shows the relationship between redshift z and *lookback time* (in billions of years).

Question 2

You may recall from Section 5.4.1 of *An Introduction to Galaxies and Cosmology* that cosmological redshifts result from the expansion of space between the time of emission of light from a source and the time that it is received on the Earth. Consider the point on the graph with $z = 5.0$. By what amount has the scale factor of the Universe increased since the light was emitted?

In this model, $z = 5.0$ corresponds to a lookback time of 12.3 billion years. The light has been travelling for this length of time through an expanding Universe, its redshift increasing as the scale factor changes, until its detection in the present day.

This is an important point to bear in mind as you look at the other graphs – the redshift depends on the time at which the light was emitted. Specifically, as the answer to Question 2 shows, redshift is related to the scale factor of the Universe at the time of emission.

- Higher redshifts correspond to earlier times of emission.
- A higher redshift implies a smaller value of the scale factor at the time of emission.

Now consider the overall shape of the lookback time graph. The graph is not a straight line, but rises steeply and then flattens off. At high redshifts ($z > 10$), the lookback time approaches the total age of the Universe.

Question 3

Based on your answer to Question 2, write a brief explanation of the main features of the lookback time graph. You should comment on the following features:

(a) The overall shape of the curve – the lookback time increases as redshift increases.

(b) The fact that the curve is not a straight line, rising steeply at low redshifts but flatttening out at high redshifts.

(It isn't necessary to explain the exact numbers – concentrate on accounting for the overall shape and features of the graph.)

The exact shape of the curve will of course depend on the amount of acceleration or deceleration of expansion and you will explore this in more detail in Section 8.

5 Distance as function of redshift

After lookback time, the next parameter to consider is the distance to a source as a function of its redshift. You can display these plots for the current model simply by selecting from the Choose graph box – there is no need to recalculate unless you change one of the model parameters. Note that there are two distance options in the Choose graph box: Distance now and Distance at emission.

■ Why is it necessary to have these two different plots of distance against redshift?

❑ As seen from the plot of lookback time against redshift, the light emitted from distant galaxies has travelled for a considerable length of time before being received on the Earth. During the time that the light has been travelling, the Universe has continued to expand, so the present-day distance will be much greater than the distance to the source when the light was emitted – more so for higher values of z.

Take a look at the Distance now plot first. As with the lookback time, it is probably not surprising that objects whose light has been redshifted by a greater amount lie at further distances from the Earth. As with lookback time, the distance is not directly proportional to redshift, but flattens off at higher redshifts.

Now switch to the Distance at emission plot. You might initially be surprised to see that the curve peaks between $z = 1$ and $z = 2$, and that the emission distances for larger redshifts start becoming smaller again! Take a moment to think about why this might be, before attempting Question 4 (remember that light received today from objects with different values of z has been travelling for different lengths of time).

Question 4

To understand the relationship between the two plots, complete Table 1 by filling in the values of $(z + 1)$, Distance now and Distance at emission for each of these specific values of redshift. You should be able to deduce a simple relationship between the two curves and the value of $(z + 1)$.

(In order to complete the table you can simply read the values from the graph, using the Scale graph slider to change the scale if necessary. Alternatively, you can use the Save to file button to create a spreadsheet file containing exact values. Note that the file will contain the data for all graphs that can be plotted. This file can be opened with StarOffice.)

Table 1 Grid for recording results for distance as a function of redshift.

z	$z + 1$	**Distance now**	**Distance at emission**
1.0			
3.0			
5.0			
7.0			
9.0			

Again, the precise shape of the distance–redshift curves will depend on the parameters of the model used, and you will investigate this further in Section 8.

6 Recession speed as function of redshift

Now select the plot Speed of recession now v z. You should see that, as expected, the speed of recession increases as a function of redshift: objects whose light is more redshifted are receding more rapidly from the Earth.

Up to now, you have probably left the Range of z values option set to 10. Before attempting Question 5, you may find it helpful to zoom in a little.

- Change this value to 5 and press the Calculate button before proceeding.

Question 5

As with the distance plots, there is a second plot: Speed of recession at emission v z. Compare the two plots by switching between them. Has the speed of recession for a given redshift increased or decreased between the time the light was emitted and the present time? What can you deduce about the rate of expansion of the Universe in this particular model – is it accelerating or decelerating?

- ■ You may also have noticed that for redshifts greater than about 2.0, the speeds of recession are greater than the speed of light! How is this possible, and how can light reach us from a galaxy with a recession speed greater than c?
- ❑ These speeds are the result of the expansion of space, and not of the movement of the galaxies *through* space.

They might more properly be thought of as a rate of increase of separation, rather than a recession speed. You may be aware that special relativity tells us that the speed of light is constant in any frame of reference: light emitted form a receding source does not travel slower than light emitted from a stationary source. Once the light has been emitted from an object, it always travels at a speed c relative to the *local* space, not relative to the source.

7 Hubble parameter as function of redshift

The next graph to consider is that of Hubble parameter against redshift. (Note that the cosmological modeller refers to the time varying Hubble parameter as the ‘Hubble constant’.) In order to get a perspective on what to expect, let’s first think about what would happen in a Universe with no acceleration or deceleration of the expansion rate.

If the rate of expansion were constant, the scale factor $R(t)$ would increase at a constant rate. So the rate of change of the scale factor $\dot{R}(t)$ would be constant.

- ■ In such a Universe what would happen to the Hubble parameter over time?
- ❑ The Hubble parameter is defined in Equation 5.21 as the rate of change of the scale factor divided by the scale factor.

  $$H(t) = \frac{\dot{R}(t)}{R(t)}$$

  Since $\dot{R}(t)$ in this scenario is constant, this implies that the Hubble constant would *decrease* over time as the scale factor increases according to

  $$H(t) \propto 1/R(t) \qquad \text{(A)}$$

However, the redshift of an object and the scale factor $R(t)$ at the time of emission are related (see the answer to Question 2)

$$R(t) \propto \frac{1}{1+z} \qquad \text{(B)}$$

So, combining Equations A and B

$$H(z) \propto (1 + z)$$

Note that we have now written H as a function of z rather than t to emphasize that we are interested in the way in which H varies with redshift rather than time.

You may wish to try sketching the graph of Hubble parameter against redshift for this situation. You should get a straight line crossing the vertical axis ($z = 0$) at its present-day value of 72 km s^{-1} Mpc^{-1}.

Question 6

In a universe with a constant rate of change of scale factor ($\dot{R}$ = constant) and a present-day Hubble constant of 72 km s^{-1} Mpc^{-1}, a distant galaxy is observed to have a redshift of $z = 10$. What was the value of the Hubble parameter at the time of emission in this model?

We have just considered how the Hubble parameter varies with redshift z in a universe which has a constant expansion rate. Now we are in a position to compare the behaviour of this type of universe with that of the WMAP model.

- Select `Hubble constant at emission v z` from the `Choose graph` box at the top left.
- (If not already selected) select a range of z from 0 to 5, and examine the graph carefully.

In light of what you have observed, consider the following.

■ Question 6 showed that, for a constant rate of expansion, the Hubble parameter would be proportional to (z + 1). If, for a given z, a model predicts a higher value for the Hubble parameter than this constant expansion model, does that imply that the rate of expansion was *greater* or *smaller* in the past than its present value?

❑ A given z corresponds to a particular value of scale factor at the time of emission. If the value of $H(z)$ at this time is higher than that predicted by the constant expansion model, this implies that the rate of expansion in the past was greater than its present value.

So if a model predicts *higher* values of $H(z)$ than the constant expansion model, this implies that the expansion has *decelerated* since the light was emitted.

Conversely, if a model gave a value of $H(z)$ that was *less* than that predicted by the constant expansion model, that would imply an *acceleration* in the rate of expansion.

Question 7

By filling in Table 2, compare the Hubble parameters predicted by the WMAP model with what you would expect from constant $\dot{R}$ model described in Question 6. What conclusions can you draw about the rate of expansion of the Universe in the WMAP model? (Again you can estimate the readings from the graph, or save the values to a spreadsheet file.)

Table 2 Grid for recording Hubble parameters predicted with different models.

z	$H(z)$/km s^{-1} Mpc^{-1}	
	constant $\dot{R}(t)$ model	**WMAP model**
0	72	
1		
2		
3		
4		
5		

8 Comparison with the critical model

Finally, let's compare the results that you have obtained with the predictions of another model mentioned in Chapter 5 – the matter-dominated critical model, which has $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$.

■ This critical model has no contribution from the cosmological constant Λ. What are the main differences that you would expect between the models?

❑ With a critical density of matter and no cosmological constant, the expansion of this Universe will continue to decelerate. The age of this Universe will be approximately 2/3 of the Hubble time and the evidence for recent accelerated expansion seen in Sections 6 and 7 should be absent from this model.

Now let's investigate the model in more detail using the cosmological calculator.

- Set $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$ and press the Calculate button.

Question 8

Repeat each of the plots that you made for the WMAP model. For each plot, note the main similarities and differences between the two models, and summarise these differences under the following headings: Lookback time, Distance, Speed and Hubble parameter.

Feel free to experiment with other combinations of parameters – you will need to keep within the constraints of a flat Universe ($(\Omega_m + \Omega_\Lambda) = 1$). Each time, make the same comparisons as you did in Question 8 and note the changes in behaviour caused by the changes in parameters.

Conclusions

During the course of this activity you have investigated how a number of cosmological parameters behave as a function of redshift. The redshift of a given object depends on the amount that the scale factor of the Universe has changed since the light observed today was emitted from the object. In all cases, the redshift is the only directly observable property and the calculated values of lookback time, distance, recession speed, and Hubble parameter all depend on the details of the cosmological model chosen.

Comparison between two different cosmological models has given differing predictions for the evolution of the Universe. The critical model predicts a consistently decelerating Universe and gives some predictions (such as the age of the Universe) that are at odds with observation.

By contrast an FRW model using values obtained from the WMAP measurements predicts a history of the Universe involving a decelerating expansion at early times and a more recent phase in which the expansion is accelerating. As you will see in Chapter 7, this description is also consistent with independent measurements of Type Ia supernovae and with the ages of stars in globular clusters. Hence this WMAP model is to be preferred over the critical model.

Answers to questions

Question 1

The value of H_0 of 72 km s^{-1} Mpc^{-1} must first be converted into units of years and Mpc.

$$H_0 = \frac{72\ \text{km s}^{-1}\ \text{Mpc}^{-1} \times 3.16 \times 10^7\ \text{s yr}^{-1}}{3.09 \times 10^{19}\ \text{km Mpc}^{-1}} \quad [\text{units: yr}^{-1}]$$

So $$\frac{1}{H_0} = \frac{3.09 \times 10^{19}\ \text{km Mpc}^{-1}}{72\ \text{km s}^{-1}\ \text{Mpc}^{-1} \times 3.16 \times 10^7\ \text{s yr}^{-1}} \quad [\text{units: yr}]$$

giving a *Hubble time* of 1.36×10^{10} years. However, in the critical model the age of the Universe is 2/3 of the Hubble time, i.e. about 9×10^9 years. This is less than the ages of stars in some globular clusters and is evidence that the critical model cannot be correct.

Question 2

The redshift is related to the scale factor by Equation 5.13 of *An Introduction to Galaxies and Cosmology*:

$$z = \frac{R(t_{\text{obs}})}{R(t_{\text{em}})} - 1$$

So, rearranging

$$\frac{R(t_{\text{obs}})}{R(t_{\text{em}})} = z + 1$$

$$\frac{R(t_{\text{em}})}{R(t_{\text{obs}})} = \frac{1}{z + 1}$$

So, at a redshift of $z = 5$, which in this model corresponds to a lookback time of 12.3 billion years,

$$\frac{R(t_{em})}{R(t_{obs})} = \frac{1}{5+1} = \frac{1}{6}$$

The scale factor when the light was emitted $R(t_{em})$ must have been one-sixth of its present-day value. The scale factor has therefore increased by a factor of six since the time of emission.

Question 3

(a) The first feature to note is that the lookback time rises continuously as a function of redshift. This is to be expected in a Universe that is expanding continuously. Higher redshifts correspond to smaller scale factors at the time of emission. Provided the scale factor is always increasing as a function of time, then – even if the rate of expansion changes – higher redshifts must correspond to earlier times of emission and hence greater lookback times.

(b) The other feature of the graph is that it rises rapidly at low redshifts and flattens off as the redshift increases. This can be understood by thinking about Equation 5.13 of *An Introduction to Galaxies and Cosmology*. A redshift of 1.0 means that the scale factor of the Universe was one-half its present value at the time of emission. From this point, it has taken 7.6 billion years for the scale factor to double in size to its present value. By contrast, the time taken for the scale factor to increase from one-eleventh to one-tenth of its present size would have been very much less than this, hence there is only a small difference in lookback times between $z = 9$ and $z = 10$.

Question 4

The completed Table 1 should look like Table 3 (note that the table shows values for distance in billions of light years and in Gpc since the *Cosmological modeller* allows you to choose your preferred unit of distance measurement).

Table 3 Results for distance as a function of redshift.

z	$z + 1$	Distance now/billion light-years*	Distance at emission/billion light-years*
1.0	2.0	10.7 (3.27)	5.33 (1.64)
3.0	4.0	20.8 (6.37)	5.20 (1.59)
5.0	6.0	25.5 (7.83)	4.25 (1.30)
7.0	8.0	28.4 (8.70)	3.54 (1.09)
9.0	10.0	30.3 (9.30)	3.03 (0.93)

*The table shows values for distance in billions of light-years and in Gpc (given in brackets).

You should be able to see that: (Distance now) = (Distance at emission) × (z + 1).

Since (z + 1) is simply a measure of how much the Universe has expanded since the light was emitted, it makes sense that the distance at emission is smaller than the distance now by that factor.

Another way of looking at it is to re-arrange Equation 5.13:

$$(z+1) = \frac{R(t_{\text{obs}})}{R(t_{\text{em}})}$$

It is important to realize that distance does increase with redshift provided that all measurements are made *at the same time*.

However, points on the distance at emission plot do *not* correspond to measurements made at the same time. Higher z values correspond to earlier emission times. For z values greater than 2 the smaller scale factor at time of emission becomes the dominant factor, making the emission distances decrease.

Question 5

You should have noticed that there is a difference in the behaviour of the curves at high redshifts and low redshifts. Above $z = 2.5$, the speed of recession today for a given object is *less* than it was at the time of emission, implying that the expansion of the Universe is slower than it was in the past. For lower redshifts $z < 2.5$, the opposite is true: the present-day speed is higher than the speed at emission, implying that the expansion is accelerating.

This apparent contradiction can be resolved by thinking about the *times of emission*. All the models discussed in Section 5.4.2 predict a decelerating expansion during the early stages of the Universe. Only at later times does the density of matter decrease sufficiently for the cosmological constant to cause the expansion to start accelerating. Thus more recent events (low z) show evidence of the acceleration, whereas at earlier times ($z > 2.5$) the Universe was still decelerating.

Question 6

In a model universe in which $\dot{R}$ is constant, the Hubble parameter is proportional to $(1 + z)$,

$$H(z) \propto (1 + z)$$

So using the fact that $H(z = 0) = H_0$, we can write

$$H(z) = H_0 \times (1 + z)$$

Using $H_0 = 72\ \text{km s}^{-1}\ \text{Mpc}^{-1}$, if $z = 10$

$$H(z = 10) = (72\ \text{km s}^{-1}\ \text{Mpc}^{-1}) \times (10 + 1) = 792\ \text{km s}^{-1}\ \text{Mpc}^{-1}$$

Question 7

The completed Table 2 is given in Table 4. The first column can be filled by using the relationship established in Question 6: that the Hubble parameter is proportional to $(z + 1)$. The second column has been filled using values from a saved spreadsheet of the results from the $\Omega_{\text{m}} = 0.27$, $\Omega_{\Lambda} = 0.73$ model.

Table 4 Hubble parameters predicted with different models.

z	$H(z)$/km s^{-1} Mpc^{-1}	
	constant $\dot{R}(t)$ model	**WMAP model**
0	72	72
1	144	122
2	216	204
3	288	306
4	360	422
5	432	553

There is again a difference in behaviour between the lower and the higher redshift ranges. For $z = 1$ and $z = 2$ the Hubble parameter is less than the constant-expansion prediction, implying an accelerating expansion. For $z = 3$ and greater, the opposite is true.

This is again consistent with the picture of a decelerating early Universe, switching over to an accelerating expansion during more recent times.

Question 8

Lookback time

The curve has the same form, rising rapidly and then flattening out – but in the critical model it reaches a lower maximum value, giving an age of about 9 billion years, or 2/3 of the Hubble time, as expected.

Distance

The distance now and distance at emission curves are still related by $(z + 1)$, but in the critical model the distances are always smaller than in the WMAP model. Again, this is to be expected since the expansion in the critical model is decelerating.

(You will see in Chapter 7 of *An Introduction to Galaxies and Cosmology* that one of the pieces of evidence for a non-zero cosmological constant is that the Type Ia supernova measurements found that the distances for a given z were *greater* than would be expected in the critical model. The WMAP model is therefore consistent with this finding.)

Speed

You should find that the two speed graphs behave differently to those in the WMAP model. In the critical model, the speed now is smaller than the speed at emission for all values of z. This indicates that the Universe is still decelerating, with even very recent emission events having lower speeds today than at the time of emission.

Hubble parameter

Again, the Hubble constant vs. z plot does not show any of the evidence of recent acceleration that you saw in Section 7. By $z = 1$, H has already increased to 200 km s^{-1} Mpc^{-1} and the whole curve lies higher than the constant expansion line.